Midterm II Practice
Math 181B, UCSD, Spring 2018

Exercise 1

Suppose $X_1, \ldots, X_n$ are i.i.d. with density

$$f_{(\mu,\sigma)}(x) = p\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2}} + (1-p)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where $p \in (0,1)$ is known, and $(\mu, \sigma) \in \mathbb{R} \times \mathbb{R}_+$ is unknown.

1. Show that the maximum likelihood estimators for μ and σ do not exist.

2. How would you estimate μ and σ? What are the asymptotic distributions of your estimates?

Exercise 2

Consider testing the null hypothesis that k Poisson means are equal,

$$H_0 : \lambda_1 = \ldots = \lambda_k \text{ against } H_1 : \text{not all means are equal},$$

using independent random samples of size n_i from Poisson distributions with means λ_i, for $i = 1, 2, \ldots, k$.
Use the likelihood ratio method to test this with asymptotic level $\alpha \in (0,1)$.

Exercise 3

Suppose that a random sample of size 30, $X_1, X_2, \cdots, X_{30}$ is drawn from a Uniform distribution on an interval $(1-\theta, 2+\theta)$. This distribution is defined through the density function

$$f_\theta(x) = \frac{1}{1+2\theta}\mathbf{1}_{(1-\theta,2+\theta)}(x).$$

1. Set up a large sample sign test (by defining a critical region or how to reject the test) for deciding whether or not the 70%-th percentile of the X -distribution is equal to 2, with $\alpha = 5\%$.

2. Given the data below, compute the exact p-value of the test in part 1.

1.42, -1.27, -0.33, -0.18, 1.56, 0.003, -0.17,-0.58, 0.44, -0.54, -1.65, -1.43, -0.08, 0.54, -1.35, 1.54, 2.30, -2.43, 0.47, 0.68

The data is represented in a histogram and qq-plot below.

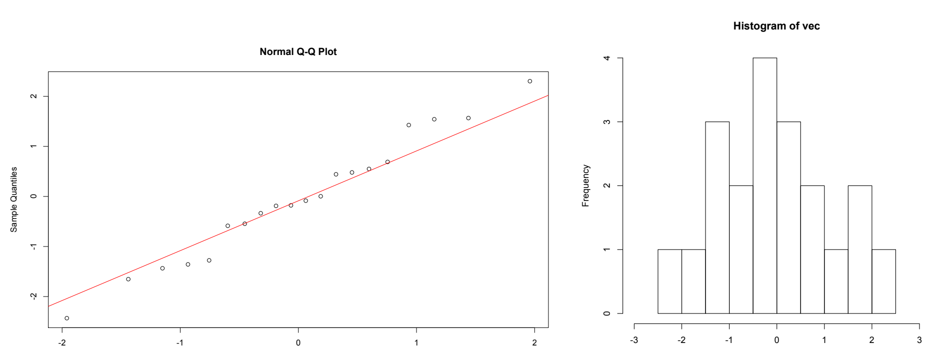

Figure 1: Data representation for part 2.

3. Compute the asymptotic p-value for the test in part 1 using the data from part 2.

4. With what probability will your procedure commit a Type II error if 3 is the true 70%-th percentile?